# A New Cosmological Model of Quintessence and Dark Matter

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We propose a new class of quintessence models in which late times oscillations of a scalar field give rise to an effective equation of state which can be negative and hence drive the observed acceleration of the universe. Our ansatz provides a unified picture of quintessence and a new form of dark matter we call *Frustrated Cold Dark Matter* (FCDM). FCDM inhibits gravitational clustering on small scales and could provide a natural resolution to the core density problem for disc galaxy halos. Since the quintessence field rolls towards a small value, constraints on slow-roll quintessence models are safely circumvented in our model.

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The recent discovery that type Ia high redshift supernovae are fainter than they would be in an Einstein-de Sitter universe suggests that the universe may be accelerating, fuelled perhaps by a cosmological constant or some other field possessing long range 'repulsive' effects [1,2]. The acceleration of the universe is related to the equation of state of matter through the Einstein equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho_c + \rho_X(1+3w_X)\right] \tag{1}$$

for cold matter $\rho_c$ and X-matter with equation of state $P_X = w\rho_X$. Clearly a necessary (but not sufficient) condition for the universe to accelerate is $w_X < -1/3$. In other words the equation of state of X-matter must violate the strong energy condition (SEC) so that $\rho_X + 3P_X < 0$. Investigations of cosmological models with $\Omega_m + \Omega_X \simeq 1$ have demonstrated that these models outperform most others in predicting the correct form for the large scale clustering spectrum, accounting for CMB anisotropies on large and intermediate angular scales and providing excellent agreement with the luminosity-distance relation obtained from observations of high redshift supernovae [3]. In addition, flat models are compelling from a theoretical viewpoint since they agree with generic predictions made by the inflationary scenario.

The literature describing phenomenological forms of matter violating the SEC is vast (see [4] for a recent review). Nevertheless two kinds of matter have been singled out in recent times as being of special interest:

1. A cosmological constant $P_X = -\rho_X$ ($w_X = -1$), $\Lambda \equiv \rho_X/8\pi G$.
2. A scalar field rolling down a potential $V(\phi)$.

For fields rolling sufficiently slowly $\ddot{\phi} \simeq 0$ and $T_{ik} \simeq V(\phi)g_{ik}$, so that $V(\phi)$ plays the role of a time-dependent $\Lambda$-term. Although appealing, models with the simplest potentials including $V \propto m^2\phi^2$ run into problems similar to those encountered by a cosmological constant. The enormous overdamping of the scalar field equation during radiation and matter dominated epochs causes $V(\phi)$ to remain unchanged virtually from the Planck epoch $z_{pl} \sim 10^{19}$ to $z \sim 2$ [5] resulting in an enormous difference in the scalar field energy density and that of matter/radiation at early times. This leads to a fine tuning problem: the relative values of $\rho_\phi$ and $\rho_m$ must be set to very high levels of accuracy $(\rho_\phi/\rho_m)_{\rm initial} \sim 10^{-123}$ in order to ensure $\rho_\phi/\rho_m \sim 1$ at precisely the present epoch.

A more reasonable assumption might be if the energy density in the $\phi$-field were comparable to that of radiation at very early times – say at the end of inflation [6]. This might even be expected if the $\phi$-field were to be an inflationary relic, its energy set by an equipartition ansatz. However for the $\phi$-field to remain subdominant until recently its energy density must decrease rapidly at early times. Such behaviour clearly cannot arise for polynomial potentials $V(\phi) \propto \phi^p$, $0 < p \lesssim few$, for which $\rho_\phi$ will rapidly dominate the total density resulting in a colossal $\Lambda$-term today if $\rho_\phi \sim \rho_{\rm rad}$ initially. Fortunately there do exist families of potentials for which the behaviour of $\rho_\phi$ is more flexible. To illustrate this, consider a minimally coupled scalar field rolling down the potential

$$V(\phi) = V_0(\cosh\lambda\phi - 1)^p. \tag{2}$$

$V(\phi)$ has asymptotic forms:

$$V(\phi) \simeq \tilde{V}_0 e^{-p\lambda\phi} \quad \text{for } |\lambda\phi| \gg 1 \ (\phi < 0), \tag{3}$$

$$V(\phi) \simeq \tilde{V}_0 (\lambda\phi)^{2p} \quad \text{for } |\lambda\phi| \ll 1 \tag{4}$$

where $\tilde{V}_0 = V_0/2^p$. Scalar field models with the potential $V(\phi) \propto e^{-p\lambda\phi}$ have the attractive property that the energy density in $\phi$ tracks the the radiation/matter component as long as the value of $\phi$ is large and negative, so that [7]:

$$\frac{\rho_\phi}{\rho_B + \rho_\phi} = \frac{3(1+w_B)}{p^2\lambda^2} \tag{5}$$

($w_B = 0,\ 1/3$ respectively for dust, radiation). During later times the form of $V(\phi)$ changes to a power law (4) resulting in rapid oscillations of $\phi$ about $\phi = 0$. The change in the form of the scalar field potential is accompanied by an important change in the equation of state of

the scalar field. As long as $V(\phi)$ is described by (3), the kinetic energy of the scalar remains larger than its potential energy $\frac{1}{2}\dot{\phi}^2 > V(\phi)$ and the scalar field equation of state mimicks background matter $w_\phi \simeq w_B$. However during the oscillatory phase $\langle\frac{1}{2}\dot{\phi}^2\rangle$ can become smaller than $\langle V(\phi)\rangle$, the virial theorem then gives the following expression for the mean equation of state [8]

$$\langle w_\phi \rangle = \left\langle \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \right\rangle = \frac{p-1}{p+1}. \tag{6}$$

The corresponding value of the scalar field density and expansion factor is given by

$$\rho_\phi \propto a^{-3(1+w_\phi)} \tag{7}$$

$$a \propto t^c,\ c = \tfrac{2}{3}(1 + \langle w_\phi \rangle)^{-1}. \tag{8}$$

From (6), (7) & (8) we find that the mean equation of state, the scalar field density and the expansion rate of the universe depend sensitively upon the value of the parameter $p$ in the potential (2). Three values of $p$ should be singled out for particular attention since they give rise to cosmologically interesting solutions:

1. $p = 1$: In this case the scalar field equation of state behaves like that of pressureless (cold) matter or dust $\langle w_\phi \rangle \simeq 0$, a scalar field potential with this value of $p$ could therefore play the role of cold dark matter (CDM) in the universe.

2. $p = 1/2$: This results in $\langle w_\phi \rangle \simeq -1/3$ and $\rho_\phi \propto a^{-2}$. This choice of the parameter $p$ leads to a 'coasting' form for the scale factor at late times: $a(t) \propto t$. A flat universe under the influence of the potential $V(\phi) = V_0(\cosh\lambda\phi - 1)^{1/2}$ will therefore have exactly the same expansion properties as an open universe without being plagued by the 'omega problem' ! (See [9] for related scenarios.)

3. Smaller values $p < 1/2$ lead to $\langle w_\phi \rangle < -1/3$. From (7) we find that the scalar field density falls off slower than either radiation ($\rho_r \propto a^{-4}$) or cold matter ($\rho_m \propto a^{-3}$). The scalar field therefore dominates the mass density in the universe at late times leading to accelerated expansion according to $a \propto t^c$, $c = 2/3(1 + \langle w_\phi \rangle) > 1$. The epoch of scalar field dominance commences at the cosmological redshift $1 + z_\star = (\Omega_\phi/\Omega_m)^{-1/3\langle w_\phi \rangle}$.

We therefore find that a scalar field with the potential (2) might serve as a good candidate for quintessence. Figure 1 confirms this by showing the density parameter $\Omega_\phi$ as a function of the cosmological scale factor. We find that the ratio $\Omega_\phi/\Omega_B$ remains approximately constant during the prolonged epoch of radiation/matter domination as the $\phi$-field tracks the dominant radiation/matter component. ($\Omega_\phi/\Omega_B < 0.2$ is necessary in order to satisfy nucleosynthesis constraints.) At the end of the matter dominated epoch $\Omega_\phi/\Omega_B$ begins to grow as the scalar field equation of state turns negative in response to rapid oscillations of $\phi$. As in earlier tracker quintessence models [6] present-day values $\Omega_\phi \simeq 0.7$, $\Omega_m \simeq 0.3$, consistent with supernovae [1,2] and other observations [3], can be obtained for a large class of initial conditions. (General potentials leading to oscillatory quintessence must satisfy $\langle V - \phi V' \rangle > 0$ where angular brackets denote the time average over a single oscillation [10]. We assume that the quintessence field couples very weakly to other matter fields so that rapid oscillations of $\phi$ do not result in particle production of the kind associated with 'preheating'.) Our potential belongs to the general category of exponential potentials which are frequently encountered in field theory [11], condensed matter physics [12] and as solutions to the non-perturbative renormalization group equations [13].

In figure 2 we compare the redshift dependence of the luminosity distance for a specific realisation of our quintessence model with $d_L$ obtained from supernovae observations.

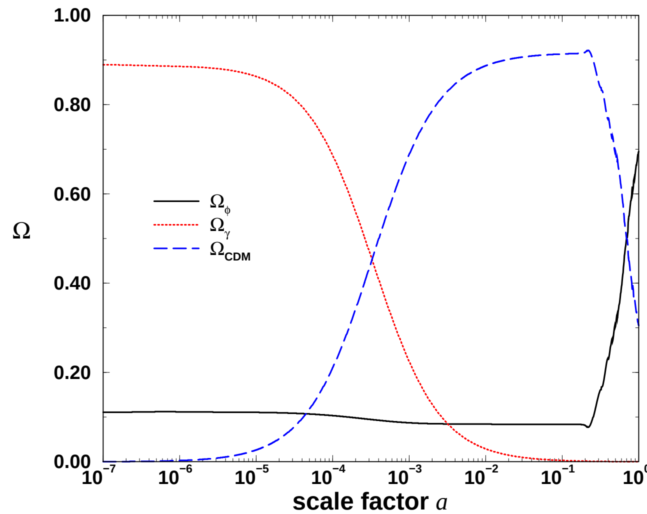


FIG. 1. The evolution of the dimensionless density parameter for the quintessence field $\Omega_\phi$ (solid line) is shown for the potential (2) with $p = 0.2$. Matter (dashed line) and radiation densities (dotted line) are also shown. For $z \gg 1$ the quintessence field satisfies the tracker solution (5) and contributes a fixed fraction to the total density of background matter/radiation. At later times $z \lesssim 2$ scalar field oscillations commence and the density in the quintessence component rapidly dominates the mass density of the universe leading to $\Omega_\phi \sim 0.7$ today. In this figure the parameters of the quintessence model used are: $\tilde{V}_0 = 8 \times 10^{-9} m_{Pl}^2 Mpc^{-2}$, $\lambda = 30 m_{Pl}^{-1}$.

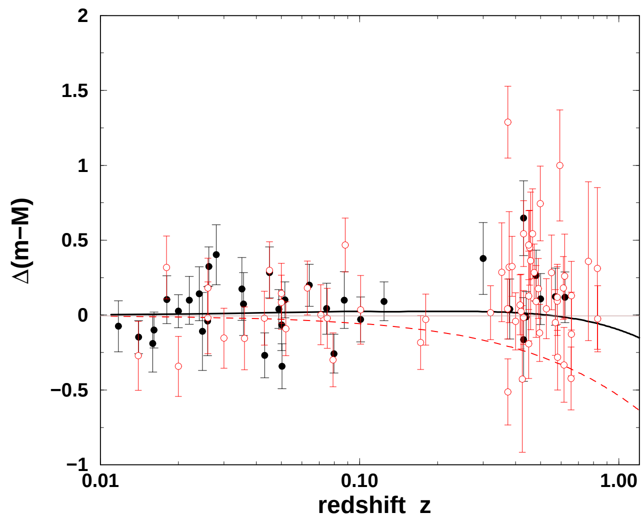


FIG. 2. The luminosity distance *vs.* redshift for the model shown in fig. 1 (solid line). The dashed line is the standard CDM model (shown for comparison) and the horizontal line corresponds to the fiducial empty Milne universe $\Omega_m \to 0$. The filled circles show supernovae data from High-Z Supernova Search Team [2] and the opaque circles show data from Supernova Cosmology Project [1]. The low-z supernovae are from the Calan-Tololo sample.

We would also like to draw attention to the possibility of a unified picture of quintessence and cold dark matter in which both components are described by a pair of scalar fields evolving under the action of the potential (2) but with different values of the exponent $p$:

$$V(\phi,\psi) = V_\phi(\cosh\lambda_\phi\phi - 1)^{p_\phi} + V_\psi(\cosh\lambda_\psi\psi - 1)^{p_\psi} \tag{9}$$

$p_\psi = 1$ in the case of CDM and $p_\phi \lesssim 0.5$ in the case of quintessence. This approach (along to the lines suggested by [14]) ameliorates the 'coincidence problem' between dark matter (CDM) and quintessence which arises in standard cosmology. It also significantly reduces the discrepancy between the present value of $\rho_m/\rho_r$ and that at the end of Inflation. Figure 3 shows a working example.

It is interesting that the CDM particle in this scenario can be ultra-light, its mass ($m^2 = V'' \sim H^2(t_*)$) is related to the epoch $t_*$ when $\psi$ begins to oscillate and its Compton wavelength $\lambda_c = m^{-1}$ can easily be of order a kilo-parsec or smaller. (In the cosmological model illustrated in figure 3 the CDM field $\psi$ begins to oscillate at $z \approx 10^5$ so that $m^{-1} \simeq 228$ parsec.) Cold dark matter made up of a condensate of ultra-light particles would be frustrated in its attempts to cluster on scales smaller than $\lambda_c$ because of the uncertainty principle, the resulting *Frustrated Cold Dark Matter* model (FCDM) might provide a natural explanation for two major difficulties faced by the standard CDM scenario. (i) The dearth of halo dwarf galaxies: the number of dwarf's in the local group is an order of magnitude smaller than predicted by N-body simulations of SCDM [15]. (ii) The discrepency between observed shapes of galaxy rotation curves and simulated dark matter halos. Recent observations of low surface brightness (LSB) galaxies show them to possess rotation curves which indicate a constant mass density in the central core region. These observations are difficult to accomodate within the SCDM model since high resolution N-body simulations of SCDM halos indicate a cuspy central density profile having the form $\rho \propto r^{-1.5}$ in the core region [16–20].

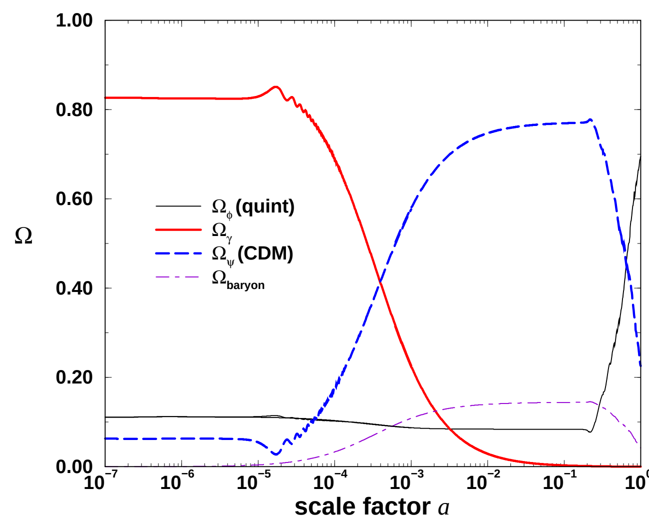


FIG. 3. The evolution of the dimensionless density parameter for the CDM field $\Omega_\psi$ (dashed line) and quintessence field $\Omega_\phi$ (thin solid line). Baryon (dash-dotted line) and radiation densities (thick solid line) are also shown. The parameters for $\phi$ have been given in the caption of figure 1. The parameters for $\psi$ are: $p_\psi = 1$, $\tilde{V}_\psi = 3 \times 10^5 m_{Pl}^2 Mpc^{-2}$, $\lambda_\psi = 8m_{Pl}^{-1}$.

It is both interesting and revealing that a physical mechanism suppressing small scale clustering arises in the FCDM model at the purely classical level. To demonstrate this we note that once $\psi$ begins oscillating $V(\psi)$ acquires the form $V(\psi) \simeq V_0[\frac{1}{2}\lambda^2\psi^2 + \frac{1}{24}\lambda^4\psi^4]$. Using the relationship [21] $\langle\dot{\psi}^2\rangle = V_0[\lambda^2\langle\psi^2\rangle + \frac{\lambda^4}{6}\langle\psi^4\rangle]$ and assuming that the gradient energy is subdominant we obtain the following expressions for the mean pressure $p_\psi$ and the mean density $\rho_\psi$: $p_\psi = \frac{V_0}{24}\lambda^4\langle\psi^4\rangle$, $\rho_\psi \simeq V_0\lambda^2\langle\psi^2\rangle$. The motion of $\psi$ is driven mainly by the quadratic term $\lambda^2\psi^2$ (which also provides the dominant contribution to the energy density). As a result $\dot{\psi} = \sqrt{V_0\lambda^2(\psi_0^2 - \psi^2)}$, which can be used to establish $\langle\psi^4\rangle = \frac{3}{2}\langle\psi^2\rangle^2$. Substituting the resulting expression for the speed of sound $v_s^2 = dp_\psi/d\rho_\psi = \rho_\psi/8V_0$ into the Jeans length $\lambda_J \simeq \sqrt{v_s^2/2\pi G\rho_\psi}$, we get

$$\lambda_J \simeq \sqrt{\frac{m_{Pl}^2}{2V_0}} \equiv \frac{\lambda}{\sqrt{2}}\left(\frac{m_{Pl}}{m}\right). \tag{10}$$

We therefore find that the Jeans length is *larger* than the Compton wavelength ($\lambda_J > m^{-1}$) if $\lambda > m_{Pl}^{-1}$ [22]. For the FCDM model illustrated in figure 3, one finds $\lambda_J \simeq 1$ kpc. A lighter particle will possess a larger Jeans length while $\lambda_J$ is smaller for a $\psi$-field which began oscillating much before $t_{eq}$, a very massive field will resemble standard CDM. By inhibiting gravitational clustering on scales smaller than $\lambda_J \sim$ kpc, FCDM is expected to give rise to galaxy halos which are less centrally concentrated leading to better agreement between theory and observations.

Finally we note that quintessence-type potentials could also arise in particle physics models which invoke the Peccei-Quinn mechanism to solve the strong CP problem in QCD. Consider for instance the following simple modification to the symmetry breaking potential responsible for the axion

$$V(\phi) = \lambda(|\phi|^2 - \frac{f^2}{2})^2 + m^2 f^2 (1 - \cos\theta)^p. \qquad (11)$$

The second term in (11) when expanded about $\theta \equiv \arg\langle\phi\rangle = 0$ acquires the form $m^2 f^2 (\theta^2/2)^p$. Accordingly rapid oscillations of the $\theta$ field about $\theta = 0$ now give rise to an equation of state described by (6), resulting in $\langle w_\theta \rangle \le -1/3$ for $p \le 1/2$. (In the standard scenario $p = 1$, $\langle w_\theta \rangle = 0$, $m \to m_a$ is the axion mass and $f \to f_{PQ}$ is the Peccei-Quinn symmetry breaking scale.) As a result an axion-like scalar with $p \le 1/2$ will be a candidate for quintessence since its energy density will diminish more slowly than that of either matter or radiation, leading to the dominance of the $\theta$-field at late times and the accelerated expansion of the universe.

It should be noted that motion under the action of potentials (2) & (11) is well defined even though for $p < 1/2$, $V'(\phi)$ is weakly singular at $\phi = 0$. One can make $V(\phi)$ mathematically more appealing by the field redefinition $\phi \to (\phi^2 + \phi_c^2)^{1/2}$, $\phi_c \to 0$, this will not affect our results in any significant way.

It is also worth mentioning that $V'' < 0$ during the oscillatory stage if $p < 1/2$. This is likely to affect very long wavelength fluctuations in $\phi$ for the potential (2) since the scalar field begins oscillating fairly recently in this case.

Finally we would like to point out an important distinction which exists between the quintessence models suggested by us and those of [6]. In our models the quintessence field $\phi$ or $\theta$ oscillates about a *small* value at late times (formally $\phi, \theta \to 0$ as $t \to \infty$). Additionally the potential $V(\phi)$ is not constraind to be flat since the field does not have to slow-roll in order to give rise to a negative equation of state. As a result quantum corrections which might be significant in quintessence models in which the scalar field rolls down a flat potential to large values $\phi \gtrsim m_{Pl}$ [23] can safely be ignored in models of the kind discussed in the present paper.

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